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**Post-Keplerian parameter to test gravitomagnetic effects in binary pulsar systems**

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We study the pulsar timing, focusing on the time delay induced by the gravitational field of the binary systems. In particular, we study the gravitomagnetic correction to the Shapiro time delay in terms of Keplerian and post-Keplerian parameters, and we introduce a new post-Keplerian parameter which is related to the intrinsic angular momentum of the stars. Furthermore, we evaluate the magnitude of these effects for the binary pulsar systems known so far. The expected magnitude is indeed small, but the effect is important *per se*.

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**INTRODUCTION**

The first binary radio pulsar (PSR B 1913 + 16) was discovered some 30 years ago by Hulse and Taylor [1]. Since those years, a great amount of work has been done, and pulsars in binary systems have proved to be celestial laboratories for testing the relativistic theories of gravity (see [2] and references therein). Indeed, up to the present day, Einstein's theory of gravity, general relativity (GR), has passed all observational tests with excellent results. However, even if the aim of the experimental relativists is always to achieve a greater precision, we must not forget that most of the tests of GR come from solar system experiments, where the gravitational field is in the "weak" regime. On the other hand, it is expected that deviations from GR can occur for the first time in the "strong" field regime: hence, the solar system experiments are inadequate to this end. On the contrary, the strong gravitational field is best tested by means of pulsars. Pulsars, which are highly magnetized rotating neutron stars, are important both for testing relativistic theories of gravity and for studying the interstellar medium, stars, binary systems, and their evolution, plasma physics in extreme conditions. As for the tests of gravity, the recent discovery of the first double pulsar PSR J0737-3039 [3,4] provided an astonishing quantity of data, which make this system a rare relativistic laboratory [5].

As we pointed out elsewhere [6], this system, in particular, and binary pulsar systems in general, could be useful for testing the so-called gravitomagnetic effects. These effects are originated by the rotation of the sources of the gravitational field, which gives rise to the presence of off-diagonal  $g_{0i}$  terms in the metric tensor. The gravitational coupling with the angular momentum of the source is indeed much weaker than the coupling with mass alone, the so-called gravitoelectric interaction. In fact, the ratio between the former and the latter can be estimated to be of the order of

$$\varepsilon = \frac{g_{0\phi}}{g_{00}} \simeq \frac{JR_S}{r^2 Mc}, \quad (1)$$

where  $R_S = 2GM/c^2$  is the Schwarzschild radius of the source,  $M$  being its mass, and  $J$  (the absolute value of) its angular momentum. At the surface of the sun, which is the most favorable place in the solar system, evaluation of Eq. (1) gives  $\varepsilon \sim 10^{-12}$ , thus evidencing the weakness of the gravitomagnetic versus the gravitoelectric interaction. The smallness of  $\varepsilon$  is the reason why, though having been suggested from the very beginning of the relativistic age [7], the experimental verification of the existence of gravitomagnetic effects has been very difficult until today (see [8] and references therein). The relevance of pulsar systems for the detection of the gravitomagnetic effects lays in the fact that the ratio (1) can be less unfavorable whenever  $r$  is approaching the Schwarzschild radius of the source: this can be the case of a source of electromagnetic (e.m.) signals orbiting around a compact, collapsed object.

As for the rotation effects in pulsar binary systems, in general, the coupling of the intrinsic angular momentum of the stars with the orbital angular momentum, and the coupling of the intrinsic angular momenta of the two stars themselves were studied, together with the corresponding precession effects [9–13]. The gravitomagnetic effects on the propagation of light in a binary system were studied by Kopeikin and Mashhoon (see [14] and references therein).

In a previous paper [6] we studied the effects of the gravitational field on e.m. pulses propagating in a binary system, and we emphasized the gravitomagnetic contribution. In doing that, we used a simplified model which considered circular orbits only. Here we generalize those results, by taking into account orbits with arbitrary eccentricity, which is a more realistic approach on the basis of the knowledge of binary systems discovered until today. In particular, we study the gravitomagnetic correction to the Shapiro contribution to the time of flight of the signals, focusing on its effect on the arrival times perceived by the experimenter on the Earth, and we introduce a new post-Keplerian (PK) parameter which is related to the intrinsic angular momentum of the stars. Finally, we evaluate the

magnitude of these effects for the binary pulsar systems known so far.

## II. THE CONTEXT: PULSARS TIMING

When studying pulsars, what is measured are the pulse arrival times at the (radio) telescope over a suitably long period of time. In fact, even though individual pulses are generally weak and have an irregular profile, a regular mean profile is obtained by averaging the received pulses over a long time. Pulses traveling to the Earth are delayed because of the dispersion caused by the interstellar medium. Besides this delay, other gravitational factors influence the arrival times of the signals emitted by a pulsar in a binary system: the strong field in the vicinity of the pulsar, the relatively weak field between the two compact objects forming the binary system and, finally, the weak field of the solar system. Consequently, the arrival time  $T_N$  of the  $N$ th pulse, as measured on the Earth, depends on a set of parameters  $\alpha_1, \alpha_2, \dots, \alpha_K$ , which include a description of the orbit of the binary system:

$$T_N = F(N, \alpha_1, \alpha_2, \dots, \alpha_K). \quad (2)$$

In particular, the set  $\alpha_1, \alpha_2, \dots, \alpha_K$  includes the Keplerian parameters, together with the so-called “post-Keplerian” parameters which describe the relativistic corrections to the Keplerian orbit of the system. Since seven parameters are needed to completely describe the dynamics of the binary system (see [15] and references therein), the measurement of any two PK parameters, besides the five Keplerian ones, allows one to predict the remaining PK parameters. For instance, if the two masses are the only free parameters, the measurement of three or more PK parameters overconstrains the system and introduces theory-dependent lines in a mass-mass diagram that should intersect, in principle, in a single point [16]. This is of course true as far as the intrinsic angular momenta are not taken into account.

It is possible to obtain a relation which links the time of arrival of a pulse on the Earth to its time of emission. More in detail, the following timing formula holds, which relates the reception (topocentric) time  $T_{\text{Earth}}$  on the Earth with the emission time  $T_{\text{pulsar}}$  in the comoving pulsar frame [17]:

$$T_{\text{pulsar}} = T_{\text{Earth}} - t_0 - \frac{D}{f^2} + \Delta_{R_o} + \Delta_{E_o} - \Delta_{S_o} - (\Delta_R + \Delta_E + \Delta_S + \Delta_A), \quad (3)$$

where  $t_0$  is a reference epoch,  $D/f^2$  is the dispersive delay (as a function of the frequency of the pulses,  $f$ ),  $\Delta_R$ ,  $\Delta_E$ ,  $\Delta_S$  are, respectively, the Roemer delay, the Einstein delay, and the Shapiro delay due to the gravitational field of the binary system (while  $\Delta_{R_o}$ ,  $\Delta_{E_o}$ ,  $\Delta_{S_o}$  are the corresponding terms due to the solar system field) and  $\Delta_A$  is the delay due to aberration.

Here we are concerned with the gravitomagnetic corrections due to the intrinsic angular momentum of the stars to the Shapiro delay  $\Delta_S$ , which we analyze in the following section.

## III. THE SHAPIRO TIME DELAY

We want to calculate the relation between the coordinate emission time  $t_e$  and the coordinate arrival time  $t_a$  (as measured at the solar system center of mass).

To this end, we consider a reference frame at rest in the center of mass of the binary system. In this reference frame, the vector pointing to the pulsar emitting e.m. signals is  $\vec{x}_1$ , while the one pointing to its companion star is  $\vec{x}_2$ ; in the following, the suffix “1” will always refer to the pulsar, and “2” to its companion. Furthermore,  $\vec{x}_b$  is the position of the center of mass of the solar system.

We use the notation of Fig. 1 for the description of the pulsar orbit. We choose a first set of Cartesian coordinates  $\{x, y, z\}$ , with origin in the center of mass of the binary system, and such that the line of sight is parallel to the  $z$  axis. Then, we introduce another set of Cartesian coordinates  $\{X, Y, Z\}$ , with the same origin: the  $X$  axis is directed along the ascending node, the  $Z$  axis is perpendicular to the orbital plane. The angle between the  $x$  and  $X$  axes is  $\Omega$ , the longitude of the ascending node, while the angle between the  $z$  and  $Z$  axes is  $i$ , the inclination of the orbital plane. Let  $\vec{x}_1 = r_1 \hat{x}_1$  be the orbit of the pulsar: it is described by

$$\hat{x}_1 = \cos(\omega + \varphi) \hat{X} + \sin(\omega + \varphi) \hat{Y}, \quad (4)$$

in terms of the argument of the periastron,  $\omega$ , and the true anomaly,  $\varphi$ . Let us pose

$$\theta \doteq \omega + \varphi, \quad (5)$$

for the sake of simplicity. Then, we use the notation

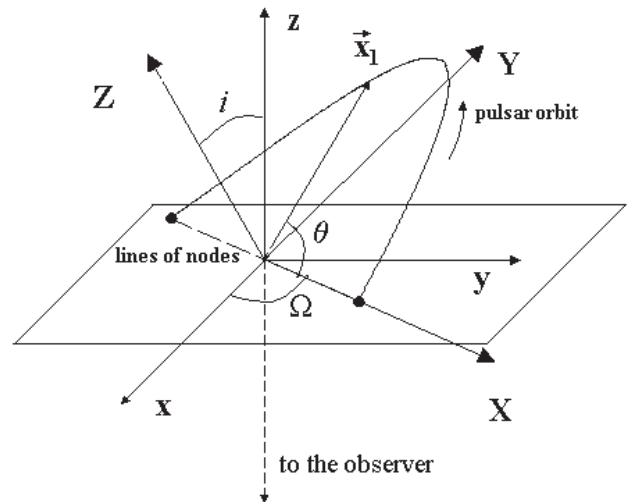


FIG. 1. Notation used for describing the pulsar orbit.

$$\vec{r} \doteq \vec{x}_1 - \vec{x}_2 \quad (6)$$

to describe the position of the pulsar with respect to its companion, and we remember that we have, for the Keplerian problem

$$r = \frac{a(1 - e^2)}{1 + e \cos \varphi}, \quad (7)$$

where  $a$  is the semimajor axis of the relative motion and  $e$  is the eccentricity. The astronomical elements  $\Omega$ ,  $i$ ,  $\omega$ ,  $a$ , and  $e$  represent the Keplerian parameters. In what follows, we will use also the definitions  $r_b = |\vec{x}_b|$ ,  $r = |\vec{r}|$ , and  $\hat{n} \doteq \vec{x}_b/r_b$ .

That being said, let us focus on the physical situation we are dealing with. If the gravitomagnetic effects are neglected, the metric describing the gravitational field of the binary system is given by (see, for instance, [18])

$$ds^2 = (1 + 2\phi)dt^2 - (1 - 2\phi)|d\vec{x}|^2, \quad (8)$$

where

$$\phi(x, y, z) = -\frac{M_1}{|\vec{x} - \vec{x}_1|} - \frac{M_2}{|\vec{x} - \vec{x}_2|}. \quad (9)$$

We see that the total gravitational potential is the sum of the two contributions, due to the pulsar, whose mass is  $M_1$ , and to its companion, whose mass is  $M_2$  [19]. Starting from (8), the mass contribution to the time delay can be evaluated following the standard approach, described, for instance, in [17]. However, we want to generalize this approach in order to take into account the effects of the rotation of the sources of the gravitational field, i.e. the gravitomagnetic effects.

To this end, we may guess that the metric, in the coordinates  $\{X, Y, Z\}$ , assumes the form

$$ds^2 = (1 + 2\phi)dt^2 - (1 - 2\phi)(dX^2 + dY^2 + dZ^2) + 4\vec{A} \cdot d\vec{X}dt, \quad (10)$$

where  $\vec{A}$  is the (total) gravitomagnetic vector potential of the system. Equation (10) generalizes the weak field metric around a rotating source (see [20,21]). We suppose that the dominant contribution to the total gravitomagnetic potential is due to the intrinsic angular momenta  $\vec{J}_1$ ,  $\vec{J}_2$  of the stars; we suppose also that  $\vec{J}_1$ ,  $\vec{J}_2$  are aligned with the total orbital angular momentum  $\vec{L}$ , i.e. perpendicular to the orbital plane. Furthermore, since we assume that the signals emitted by pulsar 1 propagate along a straight line, and because the gravitomagnetic coupling depends on the impact parameter, we may conclude that the only relevant gravitomagnetic contribution comes from the companion star 2. Consequently, the metric (10) becomes

$$ds^2 = (1 + 2\phi)dt^2 - (1 - 2\phi)(dX^2 + dY^2 + dZ^2) + 4J_2 \frac{(\vec{x} - \vec{x}_2) \cdot \hat{X}}{|\vec{x} - \vec{x}_2|^3} dYdt - 4J_2 \frac{(\vec{x} - \vec{x}_2) \cdot \hat{Y}}{|\vec{x} - \vec{x}_2|^3} dXdY. \quad (11)$$

In (11)  $|\vec{x}|$  varies along the straight line defined by

$$\vec{x}(t) = \vec{x}_1(t_e) + \frac{t - t_e}{t_a - t_e}(\vec{x}_b(t_a) - \vec{x}_1(t_e)). \quad (12)$$

If we set

$$\alpha \doteq \frac{t - t_e}{t_a - t_e}, \quad (13)$$

then  $\alpha = 0$  when  $t = t_e$ , and  $\alpha = 1$  when  $t = t_a$ . Consequently, we may write

$$\vec{x}(t) = \vec{x}_1(t_e) + \alpha(\vec{x}_b(t_a) - \vec{x}_1(t_e)). \quad (14)$$

After some straightforward manipulations, the line element (11) can be written in the form

$$ds^2 = g_{tt}dt^2 + g_{\alpha\alpha}d\alpha^2 + 2g_{t\alpha}dtd\alpha, \quad (15)$$

where

$$\begin{aligned} g_{tt} &= 1 - \frac{2M_1}{|\vec{x} - \vec{x}_1|} - \frac{2M_2}{|\vec{x} - \vec{x}_2|}, \\ g_{\alpha\alpha} &= -\left(1 + \frac{2M_1}{|\vec{x} - \vec{x}_1|} + \frac{2M_2}{|\vec{x} - \vec{x}_2|}\right)(r_b^2 + r_1^2) \\ &\quad + 2r_1r_b \sin i \sin \theta, \\ g_{t\alpha} &= -2J_2 \frac{rr_b \sin i \cos \theta}{|\vec{x} - \vec{x}_2|^3}. \end{aligned} \quad (16)$$

By setting  $ds^2 = 0$ , we easily see that the propagation time is made of three contributions, up to first order in the masses of the stars, and in the spin angular momentum:

$$\Delta t = \Delta t_0 + \Delta t_M + \Delta t_J, \quad (17)$$

where

$$\Delta t_0 = \int_0^1 \sqrt{r_b^2 + r_1^2 + 2r_b r_1 \sin i \sin \theta} d\alpha, \quad (18)$$

$$\Delta t_M = \int_0^1 \left( \frac{2M_2}{|\vec{x} - \vec{x}_2|} \right) \sqrt{r_b^2 + r_1^2 + 2r_b r_1 \sin i \sin \theta} d\alpha, \quad (19)$$

$$\Delta t_J = \int_0^1 2J_2 \frac{rr_b \sin i \cos \theta}{|\vec{x} - \vec{x}_2|^3} d\alpha. \quad (20)$$

Let us comment on (18)–(20): the first contribution is a purely geometric one, and is due to the propagation, in flat space, of e.m. signals from the pulsar toward the solar system. The second contribution represents the time depending part of the total Shapiro delay. As such it is

entirely due to the mass of the companion star; in fact it is easy to verify that the term containing the mass of the pulsar emitting e.m. signals gives a contribution which remains constant during the orbital motion. Finally, the third contribution is due to the gravitomagnetic field of the companion star. To lowest order, we obtain

$$\Delta t_0 \simeq r_b + r_1 \sin i \sin \theta, \quad (21)$$

$$\Delta t_M \simeq 2M_2 \ln \left[ \frac{2r_b}{\hat{\mathbf{n}} \cdot \vec{\mathbf{r}} + r} \right], \quad (22)$$

$$\Delta t_J \simeq \frac{2J_2 \sin i \cos \theta}{r} \frac{1 - \hat{\mathbf{n}} \cdot \hat{\mathbf{r}}}{\sin^2 i \sin^2 \theta - 1}. \quad (23)$$

The contributions (21) and (22) to the time delay are in agreement with the standard results (see [17]), while Eq. (23) gives the gravitomagnetic corrections.

From the time delay (22) we may extract the contribution which varies during the orbital motion; if we use the equation of the orbit (4) it can be written in the form

$$\Delta t_M^* = -2M_2 \ln \left[ \frac{1 - \sin i \sin(\omega + \varphi)}{1 + e \cos \varphi} \right]. \quad (24)$$

If we introduce the PK Shapiro parameters

$$\mathcal{R} \doteq M_2, \quad (25)$$

$$S \doteq \sin i, \quad (26)$$

the mass (or, gravitoelectric) contribution to the time delay becomes

$$\Delta t_M^* = -2\mathcal{R} \ln \left[ \frac{1 - S \sin(\omega + \varphi)}{1 + e \cos \varphi} \right]. \quad (27)$$

On the other hand, the gravitomagnetic contribution (23) changes continuously during the orbital motion, so that, similarly, we may write

$$\Delta t_J^* = -\frac{J_2 \sin i}{a(1 - e^2)} \left[ \frac{(\cos(\omega + \varphi))(1 + e \cos \varphi)}{1 - \sin i \sin(\omega + \varphi)} \right]. \quad (28)$$

If we introduce the PK parameter  $S$ , and define a new PK parameter

$$J \doteq J_2 \quad (29)$$

the gravitomagnetic contribution to the Shapiro time delay can be written in the form

$$\Delta t_J^* = -\frac{JS}{a(1 - e^2)} \left[ \frac{(\cos(\omega + \varphi))(1 + e \cos \varphi)}{1 - S \sin(\omega + \varphi)} \right]. \quad (30)$$

We notice that the new PK parameter  $J$  coincides with the intrinsic angular momentum of the source of the gravitomagnetic field. In particular, if we knew the rotation frequency of the source of the gravitomagnetic field (which is possible, if the latter is a visible pulsar, for instance), the

new PK parameter could give information on its moment of inertia.

The gravitoelectric and gravitomagnetic contributions to the time delay (27) and (30) can be written in the form

$$\Delta t_M^* = A_M F_M(\varphi), \quad (31)$$

$$\Delta t_J^* = A_J F_J(\varphi), \quad (32)$$

where we have introduced the following constant “amplitudes”:

$$A_M \doteq -2\mathcal{R}, \quad (33)$$

$$A_J \doteq -\frac{JS}{a(1 - e^2)}, \quad (34)$$

and the varying “phase” terms

$$F_M(\varphi) \doteq \ln \left[ \frac{1 - S \sin(\omega + \varphi)}{1 + e \cos \varphi} \right], \quad (35)$$

$$F_J(\varphi) \doteq \frac{(\cos(\omega + \varphi))(1 + e \cos \varphi)}{1 - S \sin(\omega + \varphi)}. \quad (36)$$

In particular we see that when  $S = \sin i = 1$ ,  $F_J(\varphi)$  tends to diverge as far as  $\omega + \varphi \rightarrow \pi/2$  (i.e. close to the conjunction position, when the impact parameter goes to zero).  $F_M$  too diverges in the same configuration and we see that the divergences have different “strengths,” the former being an inverse power, the latter logarithmic. The reason for that difference is easily referable to the gravitomagnetic potential affecting  $F_J$ , which has a dipolar structure, and to the monopolar gravitoelectric one affecting  $F_M$ .

Even though the divergences have no physical meaning because the actual compact objects have finite dimensions and the e.m. signals cannot pass through the center of the companion star, we see that the gravitomagnetic contribution is bigger for those systems that are seen nearly edge on from the Earth, which are the ideal candidates for revealing the gravitomagnetic effects. This is the case, for instance, of the binary system PSR J0737-3039, where, however, unfortunately the presence of a large magnetosheath zone makes the effective impact parameter much bigger than the actual linear dimension of a neutron star [6]. That being said, in the following section we give numerical estimates for the constant amplitudes  $A_M$  and  $A_J$  for the known binary pulsar systems.

#### IV. NUMERICAL ESTIMATES

The PK parameters related to the Shapiro time delay  $\mathcal{R}$ ,  $S$  have been successfully measured with great accuracy in some binary pulsar systems, such as PSR B 1913 + 16 (see [12,15]), and the recently discovered system PSR J0737-3039 (see [3–5]). Indeed, the analysis of these systems is very accurate, because of their favorable geometrical properties and, through the measurements of several PK pa-

rameters, they provided very accurate tests of GR as confronted to alternative theories of gravity. In [6], we studied the gravitomagnetic corrections to pulsar timing in a simplified situation, taking into account circular orbits only. Here, since we have generalized those results to arbitrary elliptic orbits, we may apply the formalism developed so far to all the binary pulsar systems known up to this moment, in order to estimate the magnitude of the gravitomagnetic corrections to the time delay. In Table I  $A_M$  and  $A_J$  are evaluated for the binary systems known until today [29]. A few comments on how the table has been obtained: for those systems where the available data are not complete, we have chosen for the missing data the most favorable value (see the caption of the table). Furthermore, we have estimated the intrinsic angular momentum of the sources of the gravitational field supposing that the progenitor star was only a little bigger than the sun, and that most of the angular momentum was preserved during the collapse, so that  $J_2 \approx J_\odot$ .

From Table I it is clear that the gravitomagnetic contribution is much smaller than the mass contribution, as expected. However it is possible, at least in principle, to distinguish the former from the latter, on the basis of their different dependence from the geometric parameters. In fact, from (21), (27), and (30) it is clear that the geometric and the gravitoelectric contribution are symmetric with respect to the conjunction and opposition points, while

TABLE I. Evaluation of the gravitoelectric and gravitomagnetic contributions to the time delay. For the systems PSR J1756-2251, PSR J1829 + 2456, PSR J1518 + 4904, PSR J1811-1736, and PSR B2127 + 11C, since the present data do not provide the inclination of the orbit, we chose the most favorable value for the calculations of  $A_J$ , i.e.  $\sin i = 1$ . Similarly, for the calculations of  $A_M$ , we chose the best estimate for the mass of the companion star for the systems PSR J1829 + 2456, PSR J1518 + 4904, and PSR J1811-1736, since the available data do not constrain it completely.

System	$A_M$ ( $\mu$ s)	$A_J$ (ps)
PSR B1913 + 16 <sup>a</sup>	6.9	4.2
PSR J0737-3039 <sup>b</sup>	6.2	11.8
PSR B1534 + 12 <sup>c</sup>	6.7	2.3
PSR J1756-2251 <sup>d</sup>	5.9	2.8
PSR J1829 + 2456 <sup>e</sup>	6.1	1.1
PSR J1518 + 4904 <sup>f</sup>	7.2	0.5
PSR J1811-1736 <sup>g</sup>	3.5	0.4
PSR B2127 + 11C <sup>h</sup>	6.8	6.1

<sup>a</sup>[12,15].

<sup>b</sup>[5].

<sup>c</sup>[13,22,23].

<sup>d</sup>[24].

<sup>e</sup>[25].

<sup>f</sup>[26].

<sup>g</sup>[27].

<sup>h</sup>[28].

the gravitomagnetic contribution is antisymmetric. As we pointed out in the previous paper [6], if it were possible to identify conjunction and opposition points in the sequence of arriving pulses, this fact could be exploited for extracting the gravitomagnetic effect.

Nowadays, the uncertainties in pulsar data timing are of the order of  $10^{-6}$ – $10^{-7}$  s (see for instance [5]). However, as we pointed out above, gravitomagnetic effects can become larger if the geometry of the system is favorable: in particular, when  $\sin i = 1$ ,  $F_J$  tends to diverge close to the conjunction. On the other hand, we can give an estimate of the value of the geometrical parameters needed to make  $A_J$  of the order of magnitude of the present day uncertainties. So, if we assume  $J_2 \approx J_\odot$ , in order to have  $A_J \geq 10^{-7}$  s, we must have

$$a(1 - e^2) \leq \frac{a(1 - e^2)}{\sin i} \leq 5 \times 10^4 \text{ m.} \quad (37)$$

Hence, small orbits with great eccentricity allow, at least in principle, the measurement of the gravitomagnetic effects. It might be useful, also, to estimate the rate of decay of the orbit because of the emission of gravitational waves. If we assume, for the sake of simplicity,  $e = 0$ ,  $M_1 = M_2 = 1.44M_\odot$  and that  $a$  fulfills (37), we get (see [17])

$$\left| \frac{a}{\dot{a}} \right| = \frac{5}{64} \frac{a^4}{M_1 M_2 (M_1 + M_2)} \leq 2 \times 10^7 \text{ s.} \quad (38)$$

Consequently, we may argue that these effects may become larger in the final phase of the evolution of the binary systems, i.e. during their coalescence, even though, in that phase, the weak field approach that we used in this paper would probably be rather poor, demanding for different analysis techniques.

The smallness of the gravitomagnetic delay would also require long data taking times so posing the problem of the stability of the pulsar frequency. However a peculiarity which is not blurred by any drift or noise is the physical antisymmetry of the gravitomagnetic effect, which should emerge in the long period over all other phenomena.

## V. CONCLUSIONS

In this paper, we have studied the effects of the gravitational interaction on the time delay of electromagnetic signals coming from a binary system composed of a radio pulsar and another compact object. In particular, we have focused our attention on the gravitomagnetic corrections to the time delay due to the gravitational field of the binary system (Shapiro time delay).

In doing so, we have generalized the results obtained in a previous paper, where we considered a simplified situation, taking into account circular orbits only. Here arbitrary elliptic orbits are allowed. Furthermore, by following a standard approach, we have expressed the time delay and its gravitomagnetic component in terms of Keplerian and post-Keplerian parameters.

In particular, a new post-Keplerian parameter has been introduced, which coincides with the intrinsic angular momentum of the source of the gravitational field, and could give, in some cases, information on its moment of inertia.

We have given numerical estimates of the amount of the gravitomagnetic corrections for all binary pulsar systems known until today, and we have seen that, even though they are usually much smaller than the corresponding gravito-electric ones, they can become larger for those systems that are seen nearly edge on from the Earth, which are the ideal candidates for revealing the gravitomagnetic effects in this context.

Among the known binary pulsar systems, the one having the most favorable geometrical properties for the detection of the gravitomagnetic effect is PSR J0737-3039, which, unfortunately, has a large magnetosheath that keeps the magnitude of the gravitomagnetic correction below the detectability threshold. However, we cannot exclude, at the present discovery rate of new binary pulsars, that other binary systems with favorable geometrical configurations can be found. This fact, together with the expected improvement of the sensitivity and precision of the timing of pulses, makes us cherish the hope that, in the future, it will be possible to measure the gravitomagnetic corrections to the time delay, and, in particular, the newly introduced post-Keplerian parameter.

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- [1] R.A. Hulse and J.H. Taylor, *Astrophys. J.* **195**, L51 (1975).
  - [2] I.H. Stairs, *Living Rev. Relativity* **5** (2003), <http://www.livingreviews.org/lrr-2003-5>.
  - [3] M. Burgay *et al.*, *Nature (London)* **426**, 531 (2003).
  - [4] A. Lyne *et al.*, *Science* **303**, 1153 (2004).
  - [5] M. Kramer *et al.*, in *Proceedings of the 22nd Texas Symposium on Relativistic Astrophysics, Stanford University, 2004*, eConf C041213, 0038 (2004).
  - [6] A. Tartaglia, M.L. Ruggiero, and A. Nagar, *Phys. Rev. D* **71**, 023003 (2005).
  - [7] J. Lense and H. Thirring, *Phys. Z.* **19**, 156 (1918); English translation available in B. Mashhoon, F.W. Hehl, and D.S. Theiss, *Gen. Relativ. Gravit.* **16**, 711 (1984).
  - [8] M.L. Ruggiero and A. Tartaglia, *Nuovo Cimento Soc. Ital. Fis.* **117B**, 743 (2002).
  - [9] B.M. Barker and R.F. O'Connell, *Phys. Rev. D* **12**, 329 (1975).
  - [10] B.M. Barker and R.F. O'Connell, *Phys. Rev. D* **14**, 861 (1976).
  - [11] R.F. O'Connell, *Phys. Rev. Lett.* **93**, 081103 (2004).
  - [12] J.M. Weisberg and J.H. Taylor, in *Proceedings of Radio Pulsars, Chania, Crete, 2002*, ASP Conference Series, 2003, edited by M. Bailes, D.J. Nice, and S.E. Thorsett.
  - [13] I.H. Stairs, S.E. Thorsett, and Z. Arzoumanian, *Phys. Rev. Lett.* **93**, 141101 (2004).
  - [14] S. Kopeikin and B. Mashhoon, *Phys. Rev. D* **65**, 064025 (2002).
  - [15] J.H. Taylor and J.M. Weisberg, *Astrophys. J.* **345**, 434 (1989).
  - [16] T. Damour and J.H. Taylor, *Phys. Rev. D* **45**, 1840 (1992).
  - [17] N. Straumann, *General Relativity* (Springer, Berlin, 2004).
  - [18] C.M. Will, *Theory and Experiment in Gravitational Physics* (Cambridge University Press, Cambridge, UK, 1993).
  - [19] We use units such that  $c = G = 1$ ; the signature of the space-time metric is  $(1, -1, -1, -1)$ .
  - [20] B. Mashhoon, F. Gronwald, and H.I.M. Lichtenegger, *Lect. Notes Phys.* **562**, 83 (2001).
  - [21] B. Mashhoon, gr-qc/0311030.
  - [22] I.H. Stairs, S.E. Thorsett, J.E. Taylor, and A. Wolszczan, *Astrophys. J.* **581**, 501 (2002).
  - [23] S.E. Thorsett, R.J. Dewey, and I.H. Stairs, *Astrophys. J.* **619**, 1036 (2005).
  - [24] A.J. Faulkner *et al.*, *Astrophys. J.* **618**, L119 (2005).
  - [25] D.J. Champion *et al.*, *Mon. Not. R. Astron. Soc.* **350**, L61 (2004).
  - [26] D.J. Nice, R.W. Sayer, and J.H. Taylor, *Astrophys. J.* **466**, L87 (1996).
  - [27] A.G. Lyne *et al.*, *Mon. Not. R. Astron. Soc.* **312**, 698 (2000).
  - [28] W.T.S. Deich and S.R. Kulkarni, in *Compact Stars in Binaries*, edited by J. van Paradijs, E.P.J. van den Heuvel, and E. Kuulkers (Kluwer Academic Publishers, Dordrecht, 1996).
  - [29] Notice that, in physical units we have  $A_M = GM_2/c^3$ ,  $A_J = [GJ_2 \sin i / c^4 a(1 - e^2)]$ .